NON-REGRESSION APPROACH TO DEFINING A STEM PROFILE MODEL

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INTRODUCTION

Stem profile models are essential for forestry practice as well as for studies of tree volume and growth. Tabular models, used for many years by foresters, are now being replaced by mathematical formulae that are more convenient to use but not necessarily more accurate. They do allow, however, for efficient examination of the significance of factors that influence stem taper. Developing such models usually involves accepting some reasonable assumptions underlying mathematical formulae to conform to the current paradigm for tree stem morphological line, measuring diameters along sample stems, and applying regression techniques to define model coefficients. If the model is mathematically tractable, wood volume can be used as a dependent variable in the regression model. Stem sections' volumes have been used, e.g., by Zakrzewski (1999) to define the taper model. Calibrating the integrated taper curve by defining total tree volume or the tree cumulative volume-height curve are optional solutions. Very often cross-sectional areas are used as dependent variables (see Jayaraman and Zakrzewski 1996), however, the statistical properties of volume predictions from integrated taper regression equations were found obscure (Gregoire at al. 2000). Therefore, I formulated a taper model that conforms to the common paradigm for tree form and shape in such a way that the regression techniques are not needed for calibration. This solution is described briefly in this summary.

THE MODEL FORMULATION AND CALIBRATION METHOD

It is commonly accepted that larger single-stem trees (especially conifers) are neiloidal at the bottom of the stem (butt swell), paraboloidal in the middle, and conical at the top (crown area). The top of a stem may be analysed in more detail, and it seems that in some species (e.g., jack pine) neiloidal sections occur above the base of the crown. Therefore, stem profile models should have more than one inflection point. To capture the nonlinearity of stem taper, taper models currently in use generally have a number of coefficients. Fewer coefficients, of course, may be not justified if the resulting model does not provide the required flexibility. I suggest, however, that a variable form mathematically tractable taper model with two inflection points requires only one coefficient. This parsimonious approach does not aim to achieve an elegant formula, although that is a tempting goal. The advantage of the suggested model is that it does not require use regression for calibration. It defines reasonably well tree volumes and diameters along the stem when it is calibrated using either stem analysis based data or existing tabular volume models, thus, allows for avoiding costly field data collection. The suggested one-coefficient β cross-sectional area model is a rational function:

$$ca_z = K \frac{z^2 + \beta z^3 + z^4}{z - H}$$

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where

- z is a relative location of cross-sectional area ca_i
- H is a total tree height
- K is a tree-specific constant that depends on input diameter to define ca_i and its height location z (at any height on a stem but commonly breast height). This constant may also account for tree bark thickness, e.g., when the input diameter is outside bark *DBH* and inside bark volumes and diameters are to be predicted.

The above model is a simplified form of that derived by Zakrzewski (1999), which is based on geometric premises. It is a product of two functions: one that is used to approximate the neiloidal form of a physically stable wooden column of height H, and one that is an accepted empirical modifier (third degree polynomial) used to obtain the most accurate description of tree profile. The model can be analytically integrated to define stem volume.

If tree total height, diameter at some height location, and a tree stem volume (or volume of a section of the stem) are known, the coefficient β can be calculated analytically for the tree. The mean coefficient calculated for sample trees of given species may be treated as a species-specific constant. Wood volumes, total or sectional, needed to define the model can be obtained in many ways (weight conversion, water submergence, standard formulae by Huber, Smalian, etc.). This method can be used to convert existing individual tree volume tables, commonly using tree height and *DBH* as input variables, into taper models.

RESULTS

The presented model was calibrated using the procedure presented above, and then volume and diameter predictions were examined. Three tests were completed. The first test used Scots pine (*Pinus sylvestris* L.) data described by Laasaenaho, J. (1982), the second used slash pine (*Pinus elliottii* Engelm.) data supplied by Charley Thomas (Thomas and Parresol, 1991), and the third used values from composite (all species) volume tables developed for the US Lake States by Gevorkiantz and Olsen (1955).

Test 1. Scots pine

The data consisted of total tree volumes and *DBH* measurements for 1815 Scots pine trees (above 9 m tall). Each tree was divided into 15 sections of equal length. For these trees a mean coefficient β = -1.7688 was calculated (see Fig. 1). Standard deviation of tree volume error (unbiased prediction) was 7.5%. As the diameter measurements of the stem sections were not included in the data set used for defining model coefficient , the calibrated model was sent to Finland and relative diameter prediction errors were calculated by Dr. Juha Lappi (Table 1). For a wide relative range of stem height locations diameter prediction bias was below 2%.

Table 1. Relative diameter prediction errors for Scots pine from Finland

Percent of height from ground	MEAN [%]	SEM [%]	MIN [%]	MAX [%]	SD [%]
4	-10.16	0.06	-16.25	-5.03	2.63
11	8.56	0.13	-8.16	42.95	5.39
18	2.23	0.07	-8.63	22.99	3.04
25	0.12	0.05	-9.85	14.63	2.08
32	-0.49	0.05	-10.22	8.55	1.94
39	0.01	0.06	-10.86	12.31	2.65
46	0.83	0.08	-12.24	13.01	3.47
54	1.76	0.10	-12.94	17.44	4.32
61	1.18	0.12	-18.12	22.78	5.12
68	-0.42	0.14	-23.18	19.96	6.14
75	-2.33	0.18	-38.00	24.21	7.86
82	-4.60	0.25	-45.15	34.38	10.56
89	-6.89	0.33	-52.53	54.43	13.92
96	-10.57	0.44	-69.16	89.22	18.79

Test 2. Slash pine

Data from 93 sample slash pine trees, all of them above 10 m tall, were used in this test. Tree volumes were calculated as a sum of Smalian sectional volumes. For calculating β, variable volumes were used. For some trees total tree volume was used; for other, randomly selected, only the volumes of larger sections were included. For slash pine, the mean model coefficient, β, was -1.7869. Standard deviation of volume prediction errors was 7.7%. The prediction was unbiased (mean error -1.09% with SEM = 0.89%). Standard deviation of errors of diameter predictions (all measurements) was 0.8 cm.

Table 2. Absolute diameter prediction errors for slash pine

Percent of height from ground	MEAN [cm]	SEM [cm]	MIN [cm]	MAX [cm]	SD [cm]
10	-0.30	0.06	-2.30	3.14	0.80
30	0.32	0.06	-1.73	2.45	0.87
50	0.45	0.06	-1.97	2.66	0.73
70	-0.12	0.06	-2.11	2.13	0.73
90	-0.23	0.04	-2.98	1.36	0.71

Test 3. Composite species table

Gevorkiantz and Olsen (1955) show, in tabular form, individual *DBH* and height-specific tree volumes. The authors also produced taper factors which are relative tree height-specific. Using, as an input to the model, the tabular values for inside bark diameters between 40 and 70% of total height (smallest volume prediction error), and total tree volumes for 150 different pairs of heights and dbh's, the composite coefficient β = -1.7366 was calculated. The presented model with this parameter reflects the tabular volume of the trees with a mean error of 0.08% (SEM = 0.08%), and standard deviation of errors of 1%. Regression of tree diameters on predicted diameters produced a line of intercept of -0.78 and slope of 1.01.

Table 3. Relative diameter prediction errors calculated using tables for the Lake States

Percent of height from ground	MEAN [%]	SEM [%]	MIN [%]	MAX [%]	SD [%]
10	-4.5897	0.14	-7.54	-1.27	1.71
20	-2.4366	0.06	-3.78	-1.35	0.70
30	-0.025	0.06	-1.35	1.29	0.74
40	0.2884	0.09	-1.99	3.14	1.29
50	0.2176	0.12	-2.34	4.21	1.71
60	0.6756	0.10	-3.28	3.39	1.40
70	3.6369	0.07	1.54	4.89	0.86
80	11.1405	0.17	6.09	13.21	2.13
90	27.3202	0.32	18.40	30.50	3.92

Similar to Scots pine, the smallest bias for diameter predictions was recorded for the centre section of the stems.

CONCLUSIONS

The derived model produced reasonable predictions for both tree volume and diameter of the sample species. Individual tree coefficients β declined slightly with increasing tree size, which indicates that the model needs some refining. In general, the developed method can be considered useful for most practical applications involving taper models. Better predictions can be obtained using the presented model with two coefficients (Zakrzewski 1999,).

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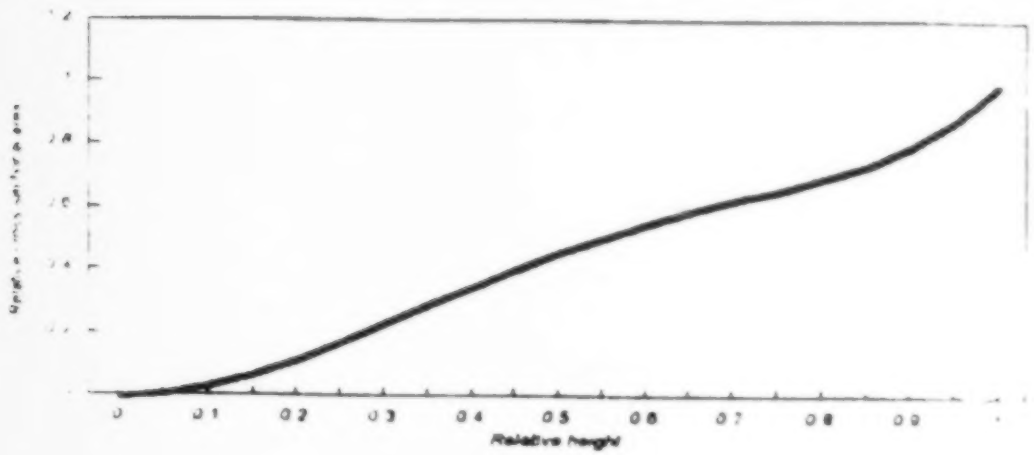

Fig. 1. Scots pine stem profile.

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